

Entropy spectrum of the apparent horizon of Vaidya black holes via adiabatic invariance

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Abstract

The spectroscopy of the apparent horizon of Vaidya black holes is investigated via adiabatic invariance. We obtain an equally spaced entropy spectrum with its quantum equal to the one given by Bekenstein [3]. We demonstrate that the quantization of entropy and area is a generic property of horizon, not only for stationary black holes, and the results also exit in a dynamical black hole. Our work also shows that the quantization of black hole is closely related to Hawking temperature, which is an interesting thing.

Keywords: Vaidya black hole, apparent horizon, entropy spectrum, adiabatic invariance

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Since the first exact solution of Einstein equation was found out, studying black holes' properties has become an important part of gravitational physics. With the discovery of laws of black hole mechanics [1–3] and Hawking radiation [4, 5], laws of black hole thermodynamics have been built up successfully, which causes deep, unsuspected connections among classical general relativity, quantum physics and statistical mechanics. Since thermodynamics is a phenomenological theory, there should exist a more fundamental theory of gravity just as statistical mechanics, then the statistic origin of black hole entropy becomes an interesting problem. In 1970's, Bekenstein proved that the quantum of the black hole horizon is given as $(\Delta A)_{min} = 8\pi l_p^2$ [3], and he also showed that the horizon area can be treated as an adiabatic invariance. In 1998, Hod [6] proposed the area spectrum $(\Delta A)_{min} = 4 \ln 3 l_p^2$ by employing the real part of quasinormal frequencies of a black hole and Bohr's correspondence principle. Later on, based on the proposal of adiabaticity of the horizon area and quasinormal frequencies, Kunstatter [7] got the entropy spectrum of a D-dimensional black hole which is the same as Hod's result. In 2008, Maggiore [8] found that when a classical black hole is perturbed, its relaxation is governed by a set of quasinormal modes with complex frequencies $\omega = \omega_R + i\omega_i$ whose behavior is the same as that of damped harmonic oscillators with real frequencies $(\omega_R^2 + \omega_I^2)^{\frac{1}{2}}$, rather than simply ω_R . Adapting the same derivation by Hod, he found that the area of the horizon of a Schwarzschild black hole is quantized in units $(\Delta A)_{min} = 8\pi l_p^2$, which is the same as Bekenstein's original result. Recently, Majhi and Vagenas [9] proposed a new approach to derive the entropy spectrum and the horizon area quantum utilizing solely the adiabaticity of black holes and the Bohr-Sommerfeld quantization rule. Later on there were many works using this method to investigate the entropy spectrum of different kinds of stationary black holes [10–24]. We think that this method is closely related to Parikh and Wilczek's tunneling method [25, 26], so it should be applied to broader circumstances. In this paper, we extend Majhi and Vagenas' method to investigate the spectroscopy of the apparent horizon of Vaidya black holes. We demonstrate that the quantization of entropy and area is a generic property of horizon, not only for stationary black holes, and the results also exist in a dynamical black hole. Our work also shows that the quantization of black hole is closely related to Hawking temperature, which is an interesting thing.

The line element of the Vaidya black hole is given by

$$ds^2 = -[1 - \frac{2M(v)}{r}]dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where $M(v)$ is the mass of the black hole and v is the advanced Eddington time coordinate. From $g_{vv} = 0$, we get the apparent horizon $r_{AH} = 2M(v)$ which is also time-like limit surface. Refs. [27, 28] calculated the Hawking temperature of the apparent horizon using tunneling method and Damour-Ruffini method, and established the thermodynamics for the apparent horizon of Vaidya black holes as follows

$$dM(v) = T_{AH}dS_{AH}, \quad (2)$$

where $T_{AH} = \frac{1}{8\pi M(v)}$ and $S_{AH} = 4\pi M^2$.

Let us consider the quantization of the apparent horizon of Vaidya black holes by applying Majhi and Vagenas' adiabatic invariance method [9]. Consider an adiabatic invariant quantity

$$I = \int p_i dq_i = \int \int_0^{p_i} dp'_i dq_i = \int \int_0^H \frac{dH'}{\dot{q}_i} dq_i = \int \int_0^H dH' d\tau + \int \int_0^H \frac{dH'}{\dot{r}} dr, \quad (3)$$

where p_i is the conjugate momentum of the coordinate q_i with $i = 0, 1$ for which $q_0 = \tau$ and $q_1 = r$. Note that we use the Euclidean time $q_0 = \tau$ and the Einstein summation convention. To get the third equation, we have used Hamilton canonical equation $\dot{q}_i = \frac{dH}{dp_i}$, where the Hamiltonian H is the total energy of the black hole. In order to calculate the adiabatic invariant quantity we shall obtain the quantity \dot{r} that appears in Eq. (3). As Ref. [9], let us consider the radial null paths. Our subsequent analysis will concentrate on the outgoing paths, since these are the ones related to the quantum mechanically nontrivial features [25]. Because τ is the Euclidean time, like Ref. [9], we use the transformation $v \rightarrow i\tau$ to Euclideanize the metric (1) and get the radial null paths,

$$\dot{r} \equiv \frac{dr}{d\tau} = \frac{i}{2}[1 - \frac{2M}{r}] = R_+(r). \quad (4)$$

Now, using Eq. (4), we have

$$\int \int_0^H dH' d\tau = \int \int_0^H dH' \frac{dr}{R_+(r)} = \int \int_0^H dH' \frac{dr}{\dot{r}}, \quad (5)$$

and the adiabatic invariant quantity (3) reads

$$I = \int p_i dq_i = 2 \int \int_0^H dH' d\tau = 2 \int \int_0^H dH' \frac{dr}{\dot{r}}. \quad (6)$$

In Ref. [9], the authors perform the τ -integration by considering the periodicity of imaginary time τ for static black holes, however we do not know if the periodicity is valid for dynamical black holes. Fortunately, we can do the r -integration. This is the main difference between Ref. [9] and this work. Using the technology in Parikh and Wilczek's tunneling method [25, 26], we can get

$$\int \int_0^H dH' \frac{dr}{\dot{r}} = \int \int_{r_{in}}^{r_{out}} \frac{dr}{\frac{i}{2}(1 - \frac{2M}{r})} dH' = \int \int_{r_{in}}^{r_{out}} \frac{2r dr}{i(r - 2M)} dH' \quad (7)$$

$$= \int_0^H 4\pi M dH' = \pi \int_0^H \frac{dH'}{\kappa}, \quad (8)$$

where $\kappa = \frac{1}{4M}$ is the surface gravity of the apparent horizon. So we obtain the adiabatic invariance,

$$I = \int p_i dq_i = 2\pi \int_0^H \frac{dH'}{\kappa} = \hbar \int_0^H \frac{dH'}{T_{AH}} = \hbar S_{AH}, \quad (9)$$

where we have used the temperature of the apparent horizon of Vaidya black holes $T_{AH} = \frac{\hbar\kappa}{2\pi}$ and the first law of thermodynamics established on the apparent horizon (2). At last, implementing the Bohr-Sommerfeld quantization rule,

$$\int p_i dq_i = nh, \quad (10)$$

we derive the entropy spectrum,

$$S_{AH} = 2\pi n, \quad (11)$$

where $n = 1, 2, 3, \dots$, and it is straightforward to see that the spacing in the entropy is given by

$$\Delta S_{AH} = S_{(n+1)AH} - S_{(n)AH} = 2\pi. \quad (12)$$

Thus, the entropy spectrum is quantized and equidistant for the apparent horizon. Recalling that in the framework of Einstein's theory of gravity, black hole entropy is proportional to the black hole horizon area [3], $S_{AH} = \frac{A}{4l_p^2}$. It is evident that if we employ the spacing of the entropy spectrum given in Eqs. (12), the quantum of the apparent horizon area has the form,

$$\Delta A = 8\pi l_p^2, \quad (13)$$

which is the same as the area quantum derived by Bekenstein [3].

Let us give some discussions. We consider a tunneling process and use the technology of the tunneling method [25, 26] which has been successfully used to calculate Hawking temperature of a variety of spacetimes, so the results of the quantization of black hole have more generality. Furthermore, our work also shows that there exists a close relationship between the entropy quantum and Hawking temperature, which reflects to some extent the viewpoint of the emergent perspective of gravity that temperature means existence of underlying degrees of freedom [30]. At last, from the calculation, it is easy to see that the ingoing path does not contribute to the adiabatic invariant quantity (3), so we only need to consider the outgoing paths.

In summary, we investigate the spectroscopy of the apparent horizon of a Vaidya black hole via adiabatic invariance and Bohr-Sommerfeld quantization rule, and obtain the quantized entropy and area spectrum which are in accordance with Bekenstein's original results [3]. These results indicate that the quantization of entropy and area of the black hole horizon is a generic property of horizon, not only for stationary black holes, and the quantization of black hole is closely related to Hawking temperature.

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